Quiz 2 solutions:

Solution 1. $y_t = m_t + \varepsilon_t$.

$m_t = a + bt + ct^2$. & $\varepsilon_t \overset{iid}{\sim} (0, \sigma^2)$.

a)

$\nabla^2 y_t = \nabla^2 m_t + \nabla^2 \varepsilon_t$.

$\nabla^2 m_t = \nabla(\nabla m_t) = \nabla(m_t - m_{t-1})$

$= \nabla m_t - \nabla m_{t-1}$

$= m_t - m_{t-1} - (m_{t-1} - m_{t-2})$

$= m_t - 2m_{t-1} + m_{t-2}$.

$\therefore \nabla^2 m_t = (a + bt + ct^2) - 2(a + bt - b + c(t-1)^2) + [a + bt - 2b + c(t-2)^2]$

$= ct^2 - 2ct^2 - 2c + 4ct + ct^2 + 4c - 4ct$.

$= 2c$. → 1 mark

$\nabla^2 \varepsilon_t = \varepsilon_t - 2\varepsilon_{t-1} + \varepsilon_{t-2}$.

$\therefore \text{Var}(\nabla^2 \varepsilon_t) = \text{Var}(\varepsilon_t) + 4\,\text{Var}(\varepsilon_{t-1}) + \text{Var}(\varepsilon_{t-2})$

$= 6\sigma^2$. — 1 mark

The series $\nabla^2 y_t$ has no trend and noise has increased. → 1 mark

b) $\nabla_2 y_t = \nabla_2 m_t + \nabla_2 \varepsilon_t$

$$\nabla_2 m_t = m_t - m_{t-2}$$
$$= a + bt + ct^2 - \left(a + b(t-2) + c(t-2)^2\right)$$
$$= a + bt + ct^2 - a - bt + 2b - ct^2 - 4c + 4ct.$$
$$= 4ct + (2b - 4c). \quad -1 \text{ mark}$$

$$\nabla_2 \varepsilon_t = \varepsilon_t - \varepsilon_{t-2}$$

$$\Rightarrow \quad \text{Var}(\nabla_2 \varepsilon_t) = 2\sigma^2. \quad -1 \text{ mark}$$

The series $\nabla_2 y_t$ has trend and is more volatile as well. -1 mark.

Solution 2. a) is incorrect because simple moving average assigns equal weight to all data points within the window, and not more weight to recent data points. – 1 mark.

b) is correct because EWMA assigns more weight to recent observations, making it more responsive to recent changes in the time series compared to a simple moving average. – 1 mark

c) is incorrect because a m.a. is a smoothing technique that reduces noise and tends to smooth out seasonality as well. Especially when series has strong seasonality, it might not provide accurate forecasts. – 1 mark

d) is incorrect as well with same reasons as c). – 1 mark.