Practice Assignment: Moments
DA 241 Statistical Foundations for Data Science
Instructor: Rhythm Grover
Teaching Assistant: Kartikay Agrawal

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1. We say that an r.v. X has a symmetric distribution about if $X-\mu$ has the same distribution as $\mu-X$. We also say that X is symmetric or that the distribution of X is symmetric. Let X be symmetric about its mean. Then show that for any odd number m, the m^{th} central moment $E(X-\mu)^m$ is 0 if it exists.

2. Let $U \sim \text{Unif}(a, b)$. Find the MGF of U.

3. Recall that the Exponential distribution is memoryless, which makes it unrealistic for, e.g., modeling a human lifetime. Remarkably, simply raising an Exponential r.v. gives rise to a Weibull distribution that improves the flexibility of Exponential. Let $X \sim \text{Expo}(\lambda)$. Define $T = X^{1/\gamma}$; $\lambda, \gamma > 0$. $T \sim \text{Wei}(\lambda, \gamma)$. The pdf of T is as follows:

$$f(t) = \gamma\lambda e^{-\lambda t^{\gamma}} t^{\gamma-1};\ t > 0$$

   Let $\lambda = 1$, $\gamma = \frac{1}{3}$.

   a) Find $P(T > s+t | T > s)$ for $s, t > 0$. Does T has memoryless property?
   b) Find the mean and variance of T, and the n^{th} moment $E(T^n)$ for $n = 1, 2, \ldots$.
   c) Determine whether or not the MGF of T exists.

4. Show that sum of independent Normals is Normal. Use the moment generating function to do so.

5. If X_1 and X_2 are independent random variables, and if X_i has the binomial distribution with parameters n_i and p, $i = 1, 2$, then $X_1 + X_2$ has the binomial distribution with parameters $n_1 + n_2$ and p.

6. Let X be a random variable with the following pmf:

$$p_X(x) = \begin{cases} \frac{6}{\pi^2 x^2} & x = 1, 2, 3, ... \\ 0 & \text{otherwise} \end{cases}$$

   Is it a valid PMF? If yes, calculate the MGF of X.

7. If X has MGF $M(t)$, what is the MGF of X? What is the MGF of $a + bX$, where a and b are constants?

8. Let $X \sim \mathbb{N}(\mu, \sigma^2)$ and $Y = e^X$. Then Y has a Log-Normal distribution (which means "log is Normal"; note that "log of a Normal" doesn't make sense since Normals can be negative). Find the mean and variance of Y using the MGF of X, without doing any integrals. Then for $\mu = 0, = 1$, find the n^{th} moment $E(Y^n)$ (in terms of n).