Practice Assignment: Expectations
DA 241 Statistical Foundations for Data Science
Instructor: Rhythm Grover
Teaching Assistant: Kartikay Agrawal

1. Let X be a random variable with the following probability mass function:

$$P_X(x) = \begin{cases} \frac{1}{2|x|(|x|+1)} & \text{if } x = \pm1, \pm2, \pm3, \ldots, \\ 0 & \text{otherwise} \end{cases}$$

   Find expectation of X.

2. An appliance has a maximum lifetime of one year. The time X until it fails is a random variable with a continuous distribution having p.d.f.:

$$f(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

3. Suppose that a point (X, Y) is chosen at random from the square S containing all points (x, y) such that $0 \leqslant x \leqslant 1$ and $0 \leqslant y \leqslant 1$. Determine the expected value of $X^2 + Y^2$.

4. Prove that if there exists a constant such that $P(X \geqslant a) = 1$, then $E(X) \geqslant a$. If there exists a constant b such that $P(X \geqslant b) = 1$, then $E(X) \geqslant b$.

5. Let us divide at random a horizontal line segment of length 5 into two parts. Let X be the length of the left hand part, write down a reasonable probability density function for X. Find the expectation of X. Prove or disprove that $E(X(5 - X)) \neq E(X)E(5 - X)$.

6. If the weather is good (which happens with probability 0.6), Lorelai walks the 2 kms to class at a speed of $V = 5$ kms per hour, and otherwise rides her motorcycle at a speed of $V = 30$ kms per hour. What is the mean of the time T to get to class? Write down the PMF of T and find its expectation.

7. Suppose there are n types of stamps, which you are collecting one by one, with the goal of getting a complete set. When collecting stamps, the stamp types are random. Assume that each time you collect a stamp, it is equally likely to be any of the n types. What is the expected number of stamps needed until you have a complete set?

8. We have a well-shued deck of n cards, labeled 1 through n. A card is a match if the card's position in the deck matches the card's label. Let X be the number of matches; Find $E(X)$.

9. In a group of n people, under the usual assumptions about birthdays, what is the expected number of distinct birthdays among the n people, i.e., the expected number of days on which at least one of the people was born? What is the expected number of birthday matches, i.e., pairs of people with the same birthday?

10. Let X be a discrete r.v. with possible values 1,2,3,. . . . Let $F(x) = P(X \leqslant x)$ be the CDF of X. Show that

$$E(X) = \sum_{n=0}^{\infty}(1 - F(n))$$