Assignment 3
DA 241 Statistical Foundations for Data Science
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1. Suppose that the binomial random variable X has parameters $n = 25$, $p = \frac{1}{4}$.

   a) Use your computer to obtain the following probabilities to 6 decimal places.
      1) $P(X = 5)$
      2) $P(X = 6)$
      3) $P(X = 7)$
      4) $P(X = 8)$

   b) Write down the probability $P(6 \leqslant X \leqslant 8)$.

   c) Give the parameters of the normal approximation to this binomial distribution indicated by the central limit theorem.

   d) Use the normal approximation to find the following probabilities, by rewriting the probabilities in part a) and b) terms of the normal random variable Y approximating the distribution of X.

2. The following data set reports the annual numbers of divorces in England and Wales for the six years between 1975 and 1980.

Year	1975	1976	1977	1978	1979	1980
Number of divorces (in thousands)	120.5	126. 7	129.1	143.7	138.7	148.3

   a) Denote the number of divorces (thousands) in any particular year by y, and the year, by x (For example, point 1, $P_1 = (x_1 = 75, y_1 = 120.5)$. Plot on the points on a scatter diagram.

   b) Observe the upward linear trend of the graph. Suppose you can model it as a straight line:

$$Y = \alpha + \beta x + W,$$

   where W is the random error component. Consider the following estimates of the slope of this line:
      1) $\hat{\beta}_1$ = the slope of the line joining the first and last points P_1 and P_6;
      2) $\hat{\beta}_2$ = the slope of the line joining the midpoint of P_1P_2 to the midpoint of P_5P_6;
      3) $\hat{\beta}_3$ = the slope of the line joining the mean of first three points to the mean of the last three points.
      4) $\hat{\beta}_4$ = least squares estimator of β
      5) $\hat{\beta}_5$ = maximum likelihood estimator of β, assuming that the error W is normally distributed with mean 0 and variance σ^2

   Check if the above estimators are unbiased and compare the variances of them in terms of σ^2. Choose one with proper justification.

3. Write down moment estimators for:

   a) the Poisson parameter λ, given a random sample $X_1, X_2, \ldots, X_n$ from a Poisson distribution;

b) the Geometric parameter p, given a random sample $X_1, X_2, \ldots, X_n$ from a geometric distribution;

c) the normal parameters μ and σ^2, given a random sample $X_1, X_2, \ldots, X_n$, from a normal distribution with unknown mean and variance;

d) the exponential parameter λ, given a random sample $X_1, X_2, \ldots, X_n$, from an exponential distribution;

e) the binomial parameter p, given a random sample $X_1, X_2, \ldots, X_n$, from a binomial distribution $B(m, p)$. (Notice the binomial parameter m here; the number n refers to the sample size. Assume m is known.)

4. A coin is flipped 100 times. Given that there were 55 heads,find the maximum likelihood estimate for the probability p of heads on a single toss. Do this using likelihood function as well as using the log-likelihood function.

5. Suppose that the lifetime of Philips brand light bulbs is modeled by an exponential distribution with (unknown) parameter. We test 5 bulbs and find they have life times of 2, 3, 1, 3, and 4 years, respectively. What is the MLE for λ?

6. Suppose our data $X_1, \ldots, X_n$, are independently drawn from a uniform distribution $U(a, b)$. Find the MLE estimate for a and b.